FORMS OF RELATIVISTIC DYNAMICS, CURRENT OPERATORS AND DEEP INELASTIC LEPTON-NUCLEON SCATTERING

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Abstract

The three well-known forms of relativistic dynamics are unitarily equivalent and the problem of constructing the current operators can be solved in any form. However the notion of the impulse approximation is reasonable only in the point form. In particular, the parton model which is the consequence of the impulse approximation in the front form is incompatible with Lorentz invariance, P invariance and T invariance. The results for deep inelastic scattering based on the impulse approximation in the point form give natural qualitative explanation of the fact that the values given by the parton model sum rules exceed the corresponding experimental quantities.

1. Forms of Relativistic Dynamics

The title of this section repeats the title of paper[1] in which the notion of the forms of relativistic dynamics was introduced for the first time. In quantum field theory the four-momentum operators $\hat{P}^\mu$ ($\mu = 0,...3$) and the representation generators of the Lorentz group $\hat{M}^{\mu\nu} = -\hat{M}^{\nu\mu}$ ($\mu,\nu = 0,...3$) are given by integrals from the energy-momentum and angular momentum tensors over a space-like (or light-like) hypersurface. Dirac[1] related the different choices of the hypersurfaces to the different forms of relativistic dynamics.

By definition, the description in the point form implies that the operators $\hat{M}^{\mu\nu}$ are the same as for noninteracting particles, and thus interaction terms can be present only in the operators $\hat{P}$. If some operator is the same as for the noninteracting system, we shall write this operator without "a hat". Therefore, the point form is defined by the condition $\hat{M}^{\mu\nu} = M^{\mu\nu}$ and in the general case $\hat{P}^\mu \neq P^\mu$ for all μ.

The description in the instant form implies that the operators of ordinary momentum and angular momentum do not depend on interactions, i.e. $\hat{\mathbf{P}} = \mathbf{P}$, $\hat{\mathbf{M}} = \mathbf{M}$ ($\hat{\mathbf{M}} = (\hat{M}^{23}, \hat{M}^{31}, \hat{M}^{12})$) and therefore interactions may be present only in $\hat{P}^0$ and the generators of the Lorentz boosts $\hat{\mathbf{N}} = (\hat{M}^{01}, \hat{M}^{02}, \hat{M}^{03})$.

In the front form with the marked z axis we introduce the + and - components of the 4-vectors as $p^+ = (p^0 + p^z)/\sqrt{2}$, $p^- = (p^0 - p^z)/\sqrt{2}$. Then we require that the

operators $\hat{P}^+, \hat{P}^j, \hat{M}^{12}, \hat{M}^{+-}, \hat{M}^{+j}$ $(j = 1, 2)$ are the same as the corresponding free operators and therefore interaction terms may be present only in the operators $\hat{M}^{-j}$ and $\hat{P}^-$. We see that the front form contains three interaction dependent generators while the point and instant forms contain four such generators. However, in the front form the operators $\hat{U}_P$ and $\hat{U}_T$ corresponding to space reflection and time reversal necessarily depend on interactions. This is clear from the relations

$$\hat{U}_P P^+ \hat{U}_P^{-1} = \hat{P}^-, \quad \hat{U}_P M^{+j} \hat{U}_P^{-1} = -\hat{M}^{-j} \quad (j = 1, 2) \tag{1}$$

and the analogous relations for the operator $\hat{U}_T$. At the same time, in the point and instant forms we can choose representations with $\hat{U}_P = U_P$ and $\hat{U}_T = U_T$.
In paper[1] the instant form was related to the hypersurface $t = 0$, the front form to the hypersurface $x^+ = 0$, and the point form to the hypersurface $t^2 - \mathbf{x}^2 < 0$, $t > 0$, but as argued by Sokolov[2] the point form should be related to the hyperplane orthogonal to the four-velocity of the system under consideration. We shall not dwell on this question but note that the choice of the hypersurface is not crucial. This follows from the results of paper[3] in which the unitary operators relating all the three forms were explicitly constructed.
Therefore *all the forms are unitarily equivalent and choosing a form is only the matter of convenience but not the matter of principle.*

2. Constructing the Current Operators

Let $\hat{J}^\mu(x)$ be the operator of the electromagnetic or weak current where x is the point in Minkowski space. The translational invariance of the current operator implies that

$$\hat{J}^\mu(x) = exp(\imath \hat{P} x) \hat{J}^\mu(0) exp(-\imath \hat{P} x) \tag{2}$$

This relation makes it possible to reduce the problem of seeking $\hat{J}^\mu(x)$ to the problem of seeking $\hat{J}^\mu(0)$. The latter should satisfy Lorentz invariance which implies that

$$[\hat{M}^{\mu\nu}, \hat{J}^\mu(0)] = -\imath(g^{\mu\rho} \hat{J}^\nu(0) - g^{\nu\rho} \hat{J}^\mu(0)) \tag{3}$$

where $g^{\mu\nu}$ is the metric tensor in Minkowski space.
If the current operator satisfies the continuity equation $\partial \hat{J}^\mu(x)/\partial x^\mu = 0$ then, as follows from Eq. (2), this equation can be written in the form

$$[\hat{J}^\mu(0), \hat{P}_\mu] = 0 \tag{4}$$

Having constructed the current operators in some form and using the unitary operators relating these forms we can construct the current operators in any form. Therefore *the forms of dynamics are equivalent not only in problems of calculating the binding energies, scattering amplitudes etc. but also in problems of describing electromagnetic or weak properties of strongly interacting systems.*

3. Electromagnetic and Weak Processes with Large Momentum Transfer

Our intuition tells that when the momentum transfer in the electromagnetic or weak process is very large then the virtual photon or W-boson interacts only with one constituent, and the process is so quick that the interaction between this constituent and the remnants of the target can be neglected. People usually believe that the mathematical expression of such a condition is $\hat{J}^\mu(x) = J^\mu(x)$ i.e. the current operator is the same as for noninteracting particles. For calculations it suffice to use this condition at $x = 0$ and we assume that, by definition, the impulse approximation (IA) is defined by the relation

$$\hat{J}^\mu(0) = J^\mu(0) \tag{5}$$

In which form Eq. (5) may be reasonable? It is important to note that *if $\hat{J}^\mu(0)$ is free in some form then $\hat{J}^\mu(0)$ is not free in the other forms since the unitary operators relating the forms are interaction dependent.*
By looking at Eq. (3) we conclude that in the instant and front forms none of the components of $\hat{J}^\mu(0)$ can be free since some of the Lorentz group generators are interaction dependent. Moreover, the choice (5) in the front form breaks also P and T invariance, since, as noted in Sec. 1, the operators $\hat{U}_P$ and $\hat{U}_T$ are interaction dependent in this form. At the same time, the choice (5) in the point form preserves Lorentz invariance, P invariance and T invariance since the corresponding operators are free in this form.
Unfortunately, the problem is not so simple if $\hat{J}^\mu(0)$ should also satisfy the continuity equation (4). Let the initial state $|i\rangle$ be the eigenstate of the four-velocity operator with the eigenvalue G' and the eigenstate of the mass operator with the eigenvalue M'. Analogously, let the final state $|f\rangle$ be the eigenstate of the four-velocity and mass operators with the eigenvalues $G"$ and $M"$ respectively. It is always possible to consider the process in the reference frame in which $\mathbf{G}" + \mathbf{G}' = 0$ (see paper[4] for details). We choose the coordinate axes in such a way that $G'_x = G'_y = 0$. Then, as follows from Eq. (4), the x and y components of the operator $\hat{J}^\mu(0)$ are not constrained by the continuity equation and the 0 and z components satisfy the relation

$$G'^0(M" - M')\langle f|\hat{J}^0(0)|i\rangle = G"^z(M" + M')\langle f|\hat{J}^0(0)|i\rangle \tag{6}$$

In deep inelastic lepton-nucleon scattering M' is the nucleon mass, and in the Bjorken limit (when $G'^0 = G"^z$ and $M" \gg M'$) the interaction of the struck quark with the remnants of the target can be neglected[5]. Then it follows from Eq. (6) that the condition (5) for all the components of the operator $\hat{J}^\mu(0)$ is compatible with the continuity equation.

4. Brief Overview of the Parton Model Sum Rules for Deep Inelastic Lepton-Nucleon Scattering

The present theory of deep inelastic lepton-nucleon scattering is based on the operator product expansion (OPE)[6], Altarelli-Parisi equations[7] and collinear expansion[5]. Many textbooks consider deep inelastic processes in the framework of the Feynman parton model[8] proposed for explaining the phenomenon of Bjorken Scaling[9]. However there exist only a very few cases when the results[5−7] agree with the parton model. This occurs when the anomalous dimensions of the Wilson coefficients are equal to zero and the momentum transfer q is so large that all higher twist effects (which are of order $(M'^2/|q^2|)^n$) and the perturbative QCD corrections (of order $\alpha_s(q^2)^n$, where $\alpha_s(q^2)$ is the QCD running coupling constant and $n = 1, 2...$) can be neglected. We shall always assume that q indeed satisfies such properties.
There exist three sum rules which agree with the parton model at large $|q^2|$. These are the sum rules for the unpolarized neutrino-nucleon deep inelastic scattering derived by Adler[10], Bjorken[11], and Gross and Llewellyn Smith[12]. The existing experimental data do not make it possible to verify the first and the second sum rules with good accuracy, and the precise data recently obtained by the CCFR collaboration[13] show that the experimental value of the Gross-Llewellyn-Smith sum S_{GLS} is smaller than the value $S_{GLS} = 6$ predicted by the parton model. The analysis of the CCFR data in papers[14,15] shows that actually $S_{GLS} = 4.90 \pm 0.16 \pm 0.16$ and taking into account the corrections of order $\alpha_s(q^2)$ and $\alpha_s(q^2)^2$ is still insufficient to explain the above deficiency.
A very nontrivial sum rule for the polarized electron-nucleon scattering was derived by Bjorken[16]. The left-hand-side of this sum rule can be written in terms of the parton model, but the right-hand-side is given not by the normalization integral for the nucleon wave function (as in the above sum rules), but by the matrix element of the axial charge operator. Therefore, the right-hand-side is determined by low-energy physics and cannot be written in terms of the parton model. One of the major problems in comparing the Bjorken sum rule with the data is the problem of extracting the first moment of the neutron polarized structure function $g_1(x)$ from the deuteron data. For this purpose it is necessary to construct the deuteron electromagnetic current operator satisfying Eqs. (2-4). Let us also note that some data on the Gross-Llewellyn Smith and Bjorken sum rules were actually obtained at not very large $|q^2|$ (a few GeV^2).
Let us now consider the sum rules which one way or another are based on the parton model.
According to the recent precise results of the NMC group[17] the experimental value of the integral defining the Gottfried sum rule[18] is equal to 0.235 ± 0.026 instead of 1/3 in the parton model.
The EMC result[19] for the first moment of the proton polarized structure function $g_1(x)$ is $\Gamma_p = 0.126 \pm 0.010 \pm 0.015$ while the Ellis-Jaffe sum rule[20] predicts $\Gamma_p^{EJ} = 0.171 \pm 0.004$. The fact that the Ellis-Jaffe sum rule gives the value considerably exceeding the corresponding experimental quantity was also confirmed

at this conference[21,22].
The most impressive results of the parton model sum rules are those concerning the quark contribution to the nucleon's momentum and spin. The first sum rule (see, for example, the discussions in the textbooks[23] says that quarks carry only a half of the nucleon momentum, and this fact is usually considered as one of those which demonstrates the existence of gluons. The second sum rule known as "the spin crisis" says that the quark contribution to the nucleon spin is comparable with zero (a detailed discussion of the spin crisis can be found, for example, in papers[24–26]), and the SMC and E143 groups[21,22] estimate this contribution as 25%. Of course, these results are not in direct contradiction with constituent quark models since the latter are successful only at low energies. Nevertheless, our experience can be hardly reconciled with the fact that the role of gluons is so high.
The above discussion gives all grounds to conclude that the values given by the parton model sum rules systematically exceed the corresponding experimental quantities.

5. Qualitative Explanation of the Deficiency in the Parton Model Sum Rules

As shown by several authors (see, for example, papers[27] and references cited therein), the parton model is a consequence of the IA (i.e. Eq. (5)) in the front form of dynamics. Meanwhile, as noted in Sec. 3, the current operator satisfying Eq. (5) in the front form does not properly commute with the Lorentz boost generators and the operators $\hat{U}_P$ and $\hat{U}_T$. Therefore *the parton model is incompatible with Lorentz invariance, P invariance and T invariance.* The question arises what is the quantitative extent of the violation of these symmetries in the parton model? To answer this question we have to compare the results obtained by using the current operator satisfying the above symmetries with the results of the parton model.
If we assume that at large $|q^2|$ the IA is valid, then, as noted in Sec. 3, the only choice is to use the IA in the point form. Let us also note that if the operator $\hat{J}^\mu(0)$ satisfies Eq. (5), it has transitions only between single-particle states and there are no Z-diagrams. In addition, the notion of the infinite momentum frame is not necessary in the point form, since the Lorentz boost generators are free in this form and the components of the Fock column are not mixed by the Lorentz boost transformations.
A detailed comparison of the results in the IA for the front and point forms have been carried out in paper[28]. Below we qualitatively explain what is the difference between the corresponding calculations.
Let us consider a system of N particles with the masses m_i $(i = 1, ...N)$. It is not important whether the number N is finite or infinite. Let $\mathbf{k}_i$ be the momenta of these particles in their c.m. frame such that $\mathbf{k}_1 + ...\mathbf{k}_N = 0$. The energy of particle i in the c.m. frame is equal to $\omega_i(\mathbf{k}_i) = (m_i^2 + \mathbf{k}_i^2)^{1/2}$, and the *free* mass of the N-body system is equal to $M_0 = \omega_1(\mathbf{k}_1) + ... + \omega_N(\mathbf{k}_N)$.

Instead of $\mathbf{k}_i$ we can introduce the variables $(\mathbf{k}_{i\perp}, \xi_i)$, where $\mathbf{k}_{i\perp}$ is the projection of $\mathbf{k}_i$ onto the plane xy and

$$\xi_i = \frac{\omega_i(\mathbf{k}_i) + k_i^z}{M_0(\mathbf{k}_1, ...\mathbf{k}_N)} \in (0,1) \tag{7}$$

For simplicity we shall suppress the spin variables in the internal wave function $\chi(\mathbf{k}_{1\perp}, \xi_1, ...\mathbf{k}_{N\perp}, \xi_N)$. Then we can choose this function in such a way that the normalization condition is

$$\int |\chi(\mathbf{k}_{1\perp}, \xi_1, ...\mathbf{k}_{N\perp}, \xi_N)|^2 \delta^{(2)}(\mathbf{k}_{1\perp} + ... + \mathbf{k}_{N\perp}) \cdot$$
$$\delta(\xi_1 + ... + \xi_N - 1) \prod_{i=1}^{i=N} d^2\mathbf{k}_{i\perp} d\xi_i = 1 \tag{8}$$

Consider the process in the reference frame where the momentum P' of the initial nucleon is such that $\mathbf{P}'_\perp = \mathbf{q}_\perp = 0$, and P'^z is positive and very large. Then the calculations in the front form[27,28] give the well-known result that if the virtual photon is absorbed by quark i then $\xi_i = x$ where $x = |q^2|/2(P'q)$ is the Bjorken variable. At the same time, the calculations in the point form[28] show that the relation between ξ_i and x is

$$M_0(\mathbf{k}_{1\perp}, \xi_1, ...\mathbf{k}_{N\perp}, \xi_N)(1 - \xi_i) = M'(1 - x) \tag{9}$$

This relation shows that the difference between the parton model and our approach depends on the difference between the free mass M_0 and the real nucleon mass M'. In the nonrelativistic approximation $M_0 = M'$ since the binding and kinetic energies can be neglected and both quantities are equal to $m_1 + ... + m_N$. However there is no reason to believe that the nucleon is the nonrelativistic system.
Equation (9) makes it possible to explicitly determine the ξ_i as a function of the other internal variables and x: $\xi_i = \xi_i(\mathbf{k}_{\imath\perp}, int_i, x)$ where int_i means the internal variables for the system $1, ...i-1, i+1, ...N$. The details are given in paper[28]. It is important to note that when $x \in [0,1]$, $\xi \in [\xi_i^{min}, 1]$ where ξ_i^{min} is the function $\xi_i(\mathbf{k}_{\imath\perp}, int_i, x = 0)$. *The important conclusion is that the data on deep inelastic lepton-nucleon scattering do not make it possible to determine the ξ_i distribution of quarks at $0 < \xi_i < \xi_i^{min}$. Of course, the quantity ξ_i^{min} can be determined only if some concrete model of the nucleon is assumed.*
Now the fact that the parton model sum rules give values exceeding the corresponding experimental quantities can be qualitatively explained as follows. In the parton model the integration over $x \in [0,1]$ can be replaced by the integration over $\xi_i \in [0,1]$, and the right-hand-side of the parton model sum rules becomes proportional to the normalization integral (8). At the same time, in our approach the integration over $x \in [0,1]$ can be transformed to the integration over $\xi_i \in [\xi_i^{min}, 1]$. Therefore the right-hand-side of the corresponding sum rules does not

contain the contribution of $\xi_i \in [0, \xi_i^{min}]$, and it is natural to expect that the corresponding integral is smaller than the normalization integral (8). The details are given in paper[28].

6. Discussion

We have proposed a simple qualitative explanation of the deficiency in the parton model sum rules. Our considerations show that the parton language is not convenient since the x distribution is not uniquely related to the quark distribution over ξ_i.

The parton model gives a clear explanation of the phenomenon of Bjorken Scaling. Let us note however that Bjorken Scaling is in fact only a consequence of the fact that the dimensionless structure functions $F_i(x, M'^2/|q^2|)$ and $g_i(x, M'^2/|q^2|)$ are not singular when $|q^2| \to \infty$. This can take place in different models; in particular this takes place in model[28].

Purists can say that only those sum rules are of significance which can be derived from the OPE, i.e. the sum rules[10−12,16] (originally these sum rules were derived assuming that the algebra of the equal time commutation relations for the current operators is the same as for the free current operators, but this is a special case of the OPE). The OPE is used for the perturbative calculation of the product of the currents entering into the expression for the hadronic tensor:

$$W^{\mu\nu} = \frac{1}{4\pi} \int e^{\imath qx} \langle N | \hat{J}^\mu(x) \hat{J}^\nu(0) | N \rangle d^4x \tag{10}$$

where $|N\rangle$ is the state of the initial nucleon. Let us note however that the OPE has been proved only in the framework of the perturbation theory which does not apply to the bound state problem. As noted above, the current operator $\hat{J}^\mu(x)$ should satisfy the relations (2-4). At the same time the nucleon state $|N\rangle$ is the eigenstate of the four-momentum operator $\hat{P}$ with the eigenvalue P' and the eigenstate of the spin operator which is composed from the operators $\hat{M}^{\mu\nu}$. Therefore the relation between $\hat{J}^\mu(x)$ and $|N\rangle$ is highly nontrivial, and it is not clear in advance whether the Wilson coefficients for the product of the currents entering into Eq. (10) can be sought by using the expansion in $\alpha_s(q^2)$ while the state $|N\rangle$ entering into the same equation is not affected by such an expansion.

Let us also note that for the unambiguous verification of the sum rules[10−12,16] they should be extracted directly from the experimental data at large $|q^2|$, not using the $|q^2|$ evolution determined from the OPE or Altarelli-Parisi equations. The above considerations show that such experimental data are crucial to solve the problem whether perturbative QCD applies to deep inelastic lepton-nucleon scattering. Our considerations also pose the problem whether the role of gluons in the deep inelastic lepton-nucleon scattering observables is so high as usually believed. Of course, this problem deserves investigation.

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